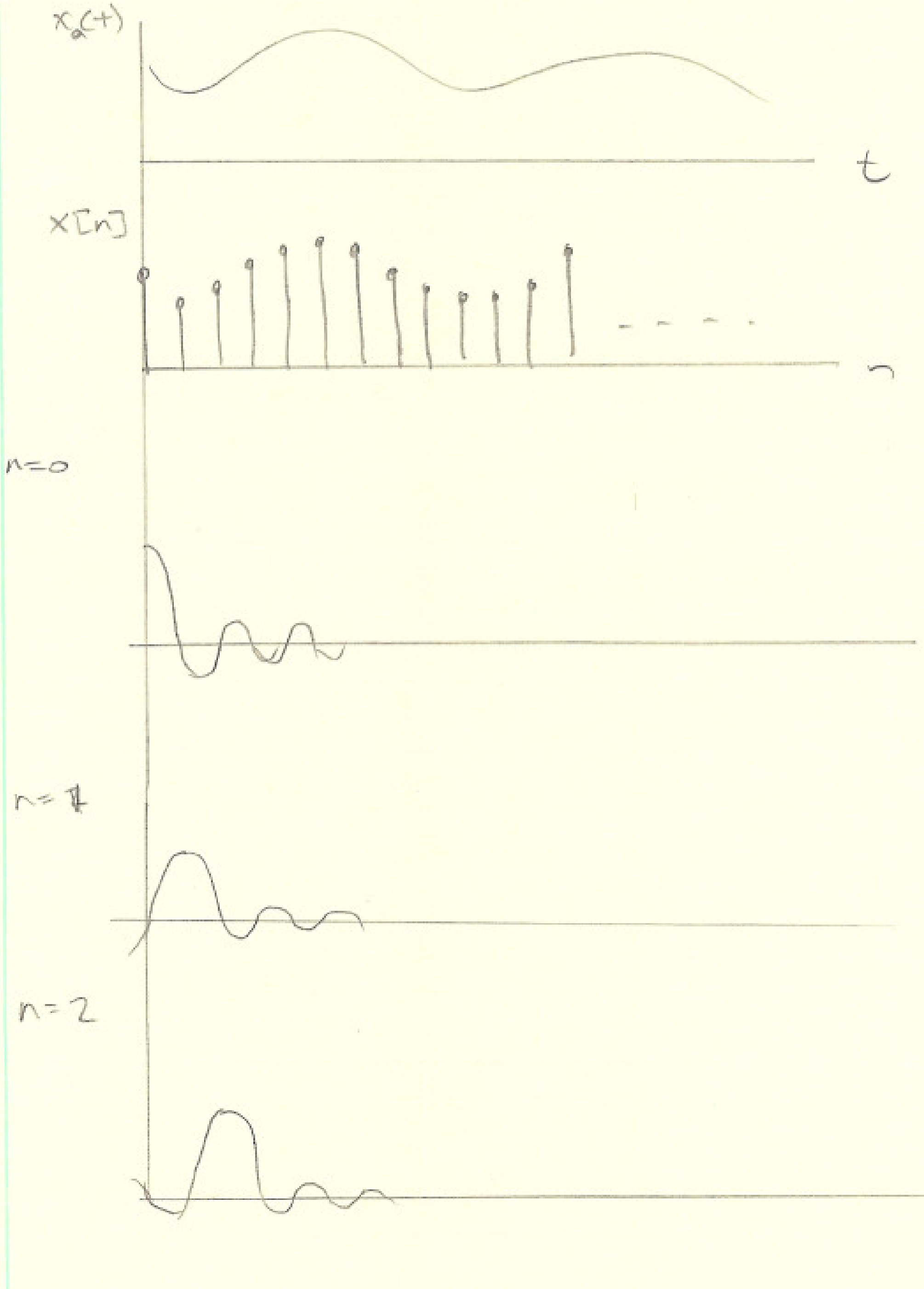
1.9.2 (cont...)

$$x_p(t) = \sum_{n=-\infty}^{\infty} x[n]\, \mathrm{sinc}\left(\frac{T - nT_s}{T_s}\right)$$

reconstructed signal.



The reconstructed signal will resemble $x_a(t)$

It should be noted that this is not realisable practically. b/c it is non-causal.

In practice D/A conversion, the ideal LPF is replaced by a practical analog LP filter and infinite order interpolation is replaced by finite order interpolation.

Several approaches are:

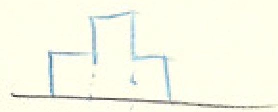
A) zero order hold (ZOH) approach

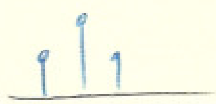
In this interpolation a given sample value is held constant until the next sample comes. This can be obtained by filtering the impulse train through a filter of the form.

$$\hat{x}_a(t) = x[n]$$

$$nT_s \le t \le (n+1)T_s$$

$$x[n] \longrightarrow \boxed{\text{ZOH}} \longrightarrow \hat{x}_a(t)$$



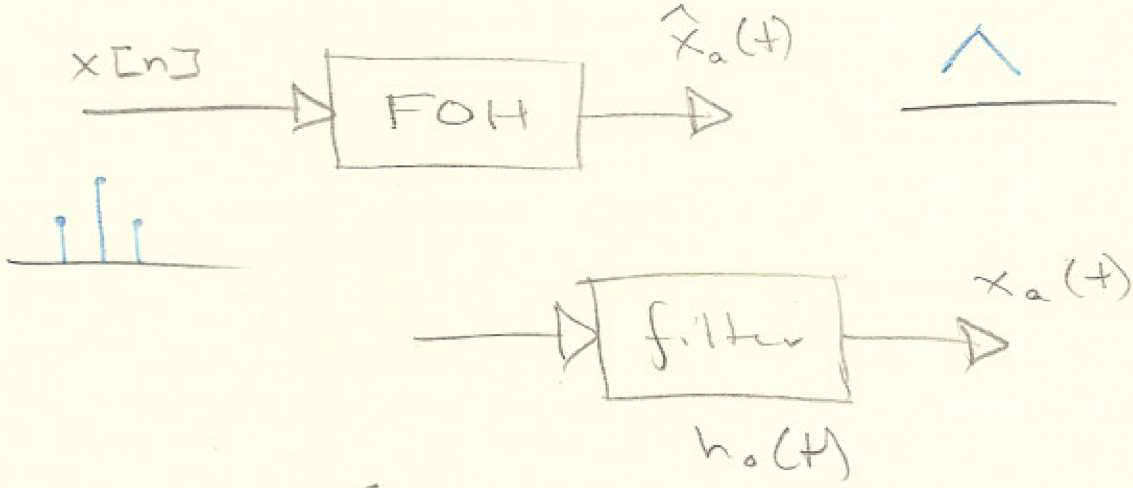


$$\longrightarrow \underset{h_o(t)}{\boxed{\text{Analog filter}}} \longrightarrow x_a(t)$$

$$x_a(t) = \hat{x}_a(t) * h_o(t)$$

$$h_o(t) = \begin{cases} 1 & 0 \le t \le T_s \\ 0 & \text{elsewhere} \end{cases}$$

B) first order hold. (FOH) interpolation



$$h_o(t) = \begin{cases} 1 + t/T_s & 0 \le t < T_s \\ 1 - t/T_s & T_s \le t \le 2T_s \\ 0 & \text{otherwise} \end{cases}$$

2. The Z transform (ch 3 of text)

Bilateral

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad (1)$$

$$= \mathcal{Z}\{x[n]\}$$

note $z = e^{j\omega}$

for z transform, the system does not have to be absolutely summable.

Unilateral

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} \qquad (2)$$

The unilateral z transform is used with regaurd to initial conditions. Assume Bilateral transform unless otherwise stated.

Note: * $z = e^{jw}$ is a complex variable.

* Region of convergence (ROC) in complex plane is all values of z for which equ ① converges

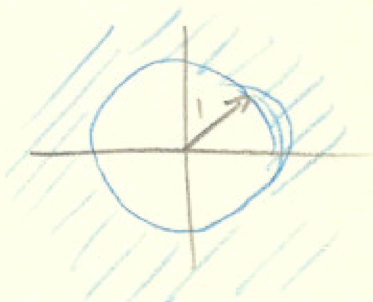
EX

$$x[n] = u[n]$$

$$X(z) = ?$$

$$= \sum_{n=-\infty}^{\infty} x[n] z^{-1}$$

$$= \sum_{n=0}^{\infty} z^{-n}$$



$$\boxed{\text{ROC } |z| > 1}$$

$$= \frac{1}{1 - z^{-1}}$$

$$= \frac{z}{z - 1}$$

EX

$$x[n] = a^n u[n]$$

$$X(z) = ?$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \frac{1}{1 - az^{-1}} = \frac{z}{z - a} =$$

$$\boxed{|a| < |z| \quad \text{ROC}}$$

2.1 Properties of z-transform.

1) linearity

$$\mathcal{Z}\{a_1 x_1[n] + a_2 x_2[n]\}$$

$$= a_1 X_1(z) + a_2 X_2(z)$$

ROC: ROC1 $\cap$ ROC2

2) shifting

$$\mathcal{Z}\{x[n-k]\} = z^{-k} X(z)$$

ROC: ROC_x except for the possible addition or delition of the origin or infinity

3.) freq shifting.

$$\mathcal{Z}\{z_0^n x[n]\} = X(z/z_0)$$

ROC: ROC_x scaled by $|z_0|$

4) folding

$$\mathcal{Z}\{x[-n]\} = X\left(\frac{1}{z}\right)$$

ROC: Inverted ROC_x

5) Complex conjugate:

$$\mathcal{Z}\{x^*[n]\} = X^*(z^*)$$

ROC: ROC_x

6) Differentiation

$$\mathcal{Z}\{n\,x[n]\} = -z\frac{dX(z)}{dz}$$

ROC: ROC_x

7) Convolution

$$\mathcal{Z}\{x_1[n] * x_2[n]\}$$

$$= X_1(z)\,X_2(z)$$

ROC: $ROC_{x_1} \cap ROC_{x_2}$